Improving the Accuracy of High Resolution Image Data Products Using Kalman Filter

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Abstract

To estimate the spacecraft attitude, Kalman filter based approach is being discussed in the paper. It is a discrete time sequential, mean square sense optimal estimator that uses the ground control point observable. In the approach, polynomial model in time is fitted for the attitude time series of the satellite. The coefficients of the polynomial thus obtained are modified at each step (ground control point observable) using an optimal Kalman filter. Thus the objective is to update the coefficients of the attitude vector and hence update the orientation parameters of the spacecraft. The approach requires *apriori* estimate of the coefficients, which can be obtained by fitting the polynomials to attitude time series. By using this approach, we had been able to achieve the desirable accuracy for the high-resolution satellite with four strips and working in step and stare mode.

Overview

1. Introduction

The utility of satellite imagery has been limited in application areas where high precision is required. This is due to the expensive and time consuming process of geometric correction. Precision corrected products are very much required in the areas where high accuracy information is required such as mapping, map updating (cartography) and change detection. For the satellite with higher orientation angles, orientation angles play a major role in the accuracy of data-products. So they need to be known accurately.

Space-borne imagery suffers from variety of geometric errors due to satellite motion, which may be due to various external forces acting on the satellite (Friedmann, et al., 1983, Caron and Simon, 1975, Wertz, 1975**)**. Digital satellite imagery thus suffers from distortions due to changing spacecraft orientation during imaging (Caron and Simon, 1975). These distortions can be removed if the attitude of the spacecraft can be estimated to sufficient precision. Generally, polynomials are fitted to the sampled attitude time series, but this fitted attitude is not sufficient to achieve required accuracy (Caron and Simon, 1975). Therefore the determination of each attitude component should be more precise and accurate.

Furthermore, there are always some deficiencies in modeling a process and therefore it becomes necessary to use the ground control points (GCP) to refine the model in case of satellite imaging. Location of the ground control point is predicted using the image to ground model. The difference in the actual location of the feature (may be by map or GPS coordinates) and the estimated ground point is used to improve the model.

In this paper we tried to estimate the spacecraft orientation parameters using Kalman filter. It is a discrete time sequential optimal estimator in mean square sense, which uses the GCP.

2. Kalman Filter Principle

It is a sequential estimator with fading memory. It generally corrects the state vector at the time of each of the observations rather than at epoch time. After the state is updated using one or more observations, it is propagated or extrapolated by a mathematical model to the time of the next update. The filter's confidence in its estimate is allowed to degrade from one update to another using the models of noise in the state vector. This causes the influence of the earlier data on the current state to fade with time so that the filter doesn't lose sensitivity to current observations (Brown and Hwang, 1997).

The optimal Kalman gain is found out by using the minimum mean square criterion and it minimizes the trace of the error covariance matrix. We wish to minimize the trace of the error covariance matrix because it is the sum of the mean square errors in the estimate of all the elements of the state vector. We can use the argument here that the individual mean square errors are also minimized when the total is minimized provided that we have enough degrees of freedom in the variation of Kalman gain, which we do in this case. We next differentiate the trace of error covariance matrix w.r.t. Kalman gain and equate it to zero for solving the optimal gain. The one gain that minimizes the mean square estimation error is called the Kalman gain.

3. Mathematical Formulations and Methodology

In the approach discussed here, first the polynomial model in time is fitted for each attitude component and the obtained coefficients are used as input to the Kalman filter. These coefficients are modified at each step (ground control point) using an optimal Kalman filter.

The basic model used to relate between the object and the image space is based on the collinearity condition that the object point, camera perspective center and corresponding image point must lie on the same line; i.e. they are collinear. Thus the image space to object space model is

$$\begin{bmatrix} f \\ -x \\ -y \end{bmatrix} = \mathbf{S.M.} \begin{bmatrix} X_A - X_S \\ Y_A - Y_S \\ Z_A - Z_S \end{bmatrix} \tag{1}$$

where, (f, -x, -y) is the image coordinate
(X_A, Y_A, Z_A) is ground coordinate
(X_S, Y_S, Z_S) is coordinate of the perspective center
M is the rotation matrix,
S is the scale factor

Calculation of the scale factor

Equation 1 can be rearranged as

$$\begin{aligned} X_A &= X_S + S^{'}L \\ Y_A &= Y_S + S^{'}M \\ Z_A &= Z_S + S^{'}N \end{aligned} \tag{2}$$

where $\begin{bmatrix} L \\ M \\ N \end{bmatrix} = M^T \begin{bmatrix} f \\ -x \\ -y \end{bmatrix}$ and $S^{'} = \frac{1}{S}$

The ground coordinate must satisfy the equation of ellipsoid

$$\frac{X_A{}^2 + Y_A{}^2}{a^2} + \frac{Z_A{}^2}{b^2} = 1 \tag{3}$$

where (X_A, Y_A, Z_A) is ground coordinate**.**

Putting the values of X_A, Y_A and Z_A in Equation 3, we get quadratic equation in S'. Solving this equation, we can calculate the scale factor.

M is the attitude dependant component in the image to ground transformation. Let the n^{th} order polynomial be fitted to each attitude component.

Let,

Roll coefficients be $\alpha_0, \alpha_1, \alpha_2, \alpha_3, \ldots \alpha_n$

Pitch coefficients be $\beta_0, \beta_1, \beta_2, \beta_3, \ldots \beta_n$

Yaw coefficients be $\gamma_0, \gamma_1, \gamma_2, \gamma_3, \ldots \gamma_n$

The basic equation can be written as

$$\begin{bmatrix} X_A \\ Y_A \\ Z_A \end{bmatrix} = \begin{bmatrix} X_S \\ Y_S \\ Z_S \end{bmatrix} + \mathbf{S'M^T} \begin{bmatrix} f \\ -x \\ -y \end{bmatrix}$$

Let us assume $G_a = \begin{bmatrix} X_A \\ Y_A \\ Z_A \end{bmatrix}$ and $G_m = \begin{bmatrix} X_S \\ Y_S \\ Z_S \end{bmatrix} + \mathbf{S'M^T} \begin{bmatrix} f \\ -x \\ -y \end{bmatrix}$

where, G_a is the actual ground coordinate and G_m be the measured coordinate of the corresponding ground control point. This is a non-linear equation, which can be linearized using the Taylor series expansion. Therefore partial derivatives of Gm are calculated with respect to attitude coefficients.

The process here is assumed to be Gaussian stochastic process.

Hence,

$$G_a(t) = G_m(t) + v(t)$$

where, v is the additive white Gaussian observation noise. We assume that equal uncertainties are associated with each components of G_a.

The coefficients obtained by fitting least square polynomial are used as the initial estimate. Let the state vector be R. Thus R is 3n x 1 matrix of coefficients which forms the state vector for the filter.

The filter equations (Tommaselli and Tozzi, 1996) are

$H=H(t_{k+1/k})=H(R(k), t_{k+1})$
$P(k+1)=P(k)-P(k)H^T[H.P(k).H^T + V]^{-1}H.P(k)$
$F(k+1)=P(k+1).H^T.V^{-1}$
$R(k+1)=R(k)+F(k+1)[G_a(t_{k+1})-G_m(R(k),t_{k+1})]$

Where, H is the matrix of partial derivatives
P is the error covariance matrix
F is the Kalman gain
V is noise covariance matrix of v
R is the state vector (coefficient matrix)
k means at kth GCP, t_k is the time at the kth GCP

4. Implementation Approach

The inputs required are

(1) The attitude time series vector.
(2) Ephemeris time series vector in ECI (Earth Centered Inertial) frame of reference.
(3) Ground coordinates of the GCP in ECI.
(4) Image coordinates of the corresponding GCP.

For a satellite with high-resolution imaging by four CCDs and working in step and stare mode, a scene was selected. A few GCPs were identified in the scene. Now

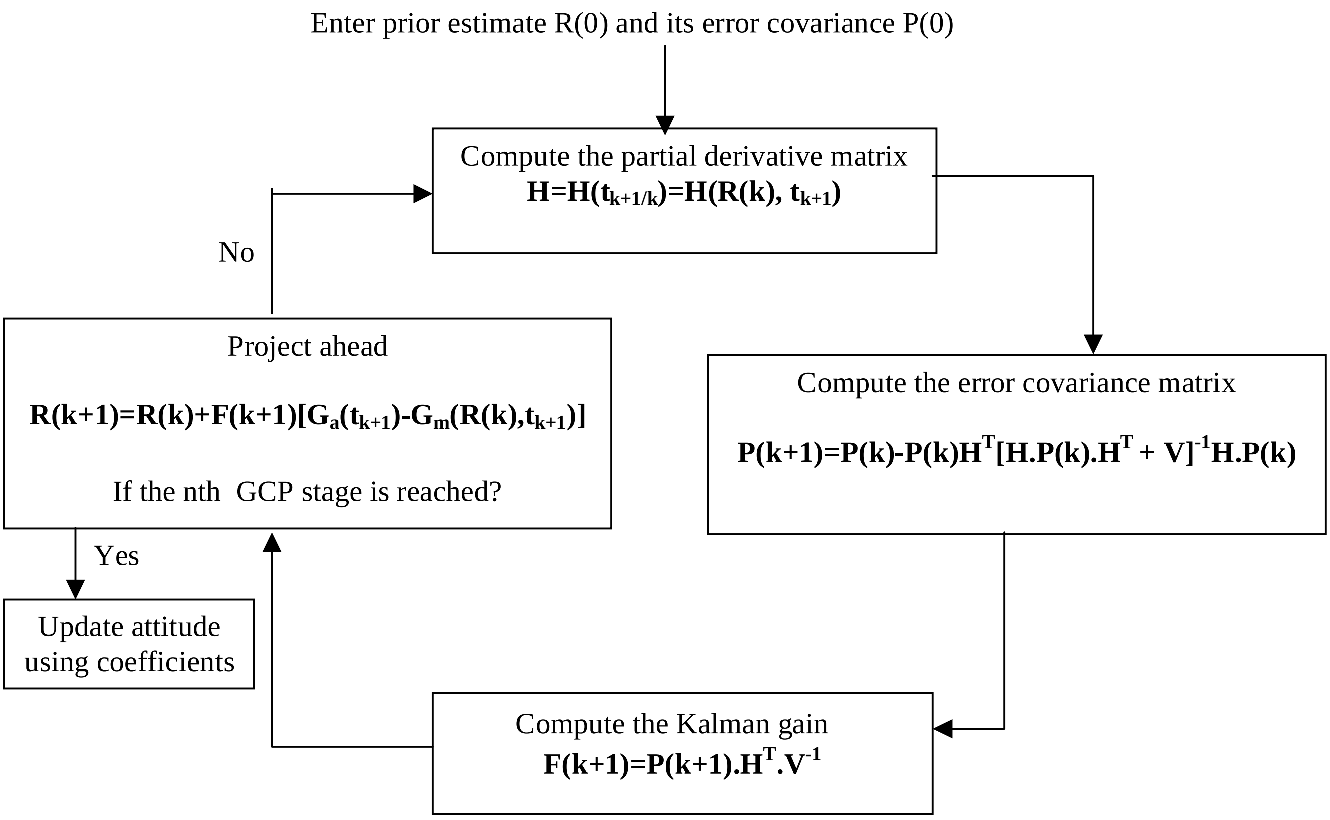

Figure 1 : Kalman Filter Loop

since the GCP coordinates are known in ECEF (Earth Centered Earth Fixed) frame of reference, they are first converted to ECI frame using the sidereal angle. Their imaging times were determined by using the relation

GCP imaging time = scene start time + (scan line*integration time)

Thus at these times, interpolated attitude at GCP time are calculated. For the whole scene, attitude time series sampled data is fitted with a second order least square polynomial. It is observed that each attitude component has a second order behavior. Therefore, it has been decided to fit a second order least square polynomial. Thus initial state vector could be written as

$$R(0) = [\alpha_0,\ \ \alpha_1,\ \ \alpha_2,\ \ \beta_0,\ \ \beta_1,\ \ \beta_2,\ \ \gamma_0,\ \ \gamma_1,\ \ \gamma_2]^T$$

where,

$\alpha_0, \alpha_1, \alpha_2$, are the roll coefficients

$\beta_0, \beta_1, \beta_2$, are the pitch coefficients

$\gamma_0, \gamma_1, \gamma_2$, are the yaw coefficients.

It is assumed that each component in state vector is uncorrelated. Uncertainty in bias value is taken to be 0.02 degrees to determine the error covariance matrix P(0). Matrix P(0) is taken to be a diagonal matrix based upon the above assumption. It is also assumed that equal uncertainties are there in each component of G_a and that they are uncorrelated. Thus the noise covariance matrix is also a diagonal matrix. Based on the measurements, the GCP accuracy is taken as 10 m. Hence V matrix is

$$[V]_{ii} = 10*10,\ \ i=1,2,3$$

These values are passed to the Kalman filter equations and at the n^{th} GCP, we get the modified polynomial coefficients. (See Fig. 1.) Using these coefficients the attitude values are regenerated at the sampled times. Thus we have updated attitude values. These updated values are used in image to ground transformation to obtain the refined ground coordinates. (Refer to Equation. 1).

5. Results and Discussions

The tables given at the end show the mean, rms and standard deviation in error in estimation while doing image to ground analysis both pre and post application of Kalman filter. The mean error in latitude estimation is 0.013694 deg (1.36 km) and that in longitude estimation is 0.02616 deg (2.61 km) before applying Kalman filter (Table 1) and after applying Kalman filter these values seem to show considerable improvement (Tables 2, 3, 4 and 5). It is also observable from Tables that by using one GCP itself, the error in estimation has been brought down drastically.

Figure 2 shows that the position estimation error goes down from 3.5 km to the order of 30 m as the number of

GCPs are increased, which is also a performance characteristic (Euclidean distance between the actual and the estimated location) of the filter and shows the stability of the filter.

6. Conclusion

From the results one can make an easy conclusion that Kalman filter can be applied successfully for the improvement of location accuracy in high-resolution imagery. In the selected scene the position estimation error has come down to 30 m compared to the initial error of 3.5 km. It is also clear from the results that this method can be applied to the images where only a few GCPs are available.

7. Acknowledgements

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8. References

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Table 1: Image to Ground Analysis before applying Kalman filter

Latitude (deg) Estimated	Longitude (deg) Estimated	Height (m) Estimated	Latitude (deg) Estimation Error	Longitude (deg) Estimation Error
23.058964	72.681744	0.275084	0.013856	0.026416
23.058235	72.650851	0.275068	0.013655	0.026309
23.057343	72.658103	0.275047	0.013497	0.026357
23.042627	72.657300	0.274715	0.013563	0.026010
23.035992	72.658850	0.274566	0.013598	0.025810
23.032825	72.663541	0.274494	0.013495	0.025809
23.027746	72.658658	0.274380	0.013414	0.025852
23.010960	72.657264	0.274002	0.013460	0.025636
23.078512	72.648963	0.275525	0.014368	0.026917
23.077914	72.653453	0.275512	0.014036	0.026557

	Latitude (deg)	Longitude (deg)
Mean Error	0.013694	0.026167
RMS Error	0.013697	0.026170
Std. Dev.	0.000305	0.000407

Table 2- Image to Ground Analysis after applying Kalman filter (Using 1 GCP)

	Latitude (deg)	Longitude (deg)	Height (m)	Latitude (deg)	Longitude (deg)
	Estimated	**Estimated**	**Estimated**	**Estimation Error**	**Estimation Error**
Model Point	23.073155	72.705247	0.275404	-0.000335	0.002913
Check Point	23.072069	72.674354	0.275380	-0.000179	0.002806
Check Point	23.070939	72.681553	0.275354	-0.000099	0.002907
Check Point	23.055910	72.680408	0.275015	0.000280	0.002902
Check Point	23.049211	72.681828	0.274864	0.000379	0.002832
Check Point	23.046026	72.686420	0.274792	0.000294	0.002930
Check Point	23.040927	72.681749	0.274677	0.000233	0.002761
Check Point	23.024916	72.680354	0.274316	-0.000496	0.002546

	Latitude (deg)	Longitude (deg)
Mean at check points	0.000059	0.002812
RMS at check points	0.000305	0.002815
Std. Dev at check points	0.000323	0.000132

Table 3: Image to Ground Analysis after applying Kalman filter (Using 2 GCPs)

	Latitude (deg)	Longitude (deg)	Height (m)	Latitude (deg)	Longitude (deg)
	Estimated	**Estimated**	**Estimated**	**Estimation Error**	**Estimation Error**
Model Point	23.073075	72.706558	0.275403	-0.000255	0.001602
Model Point	23.071991	72.675667	0.275378	-0.000101	0.001493
Check Point	23.070860	72.682865	0.275353	-0.000020	0.001595
Check Point	23.055830	72.681712	0.275013	0.000360	0.001598
Check Point	23.049131	72.683128	0.274862	0.000459	0.001532
Check Point	23.045947	72.687718	0.274790	0.000373	0.001632
Check Point	23.040848	72.683045	0.274675	0.000312	0.001465
Check Point	23.024836	72.681642	0.274314	-0.000416	0.001258

	Latitude (deg)	Longitude (deg)
Mean at check points	0.000178	0.001514
RMS at check points	0.000353	0.001519
Std. Dev at check points	0.000334	0.000138

Table 4- Image to Ground Analysis after applying Kalman filter (Using 5 GCPs)

	Latitude (deg)	Longitude (deg)	Height (m)	Latitude (deg)	Longitude (deg)
	Estimated	Estimated	Estimated	Estimation Error	Estimation Error
Model Point	23.073239	72.707479	0.275406	-0.000419	0.000681
Model Point	23.072154	72.676590	0.275382	-0.000264	0.000570
Model Point	23.071023	72.683787	0.275356	-0.000183	0.000673
Model Point	23.055991	72.682628	0.275017	0.000199	0.000682
Model Point	23.049291	72.684041	0.274866	0.000299	0.000619
Check Point	23.046106	72.688629	0.274794	0.000214	0.000721
Check Point	23.041006	72.683954	0.274679	0.000154	0.000556
Check Point	23.024991	72.682545	0.274318	-0.000571	0.000355

	Latitude (deg)	Longitude (deg)
Mean at check points	-0.000068	0.000607
RMS at check points	0.000363	0.000564
Std. Dev at check points	0.000437	0.000183

Table 5- Image to Ground Analysis after applying Kalman filter (Using 8 GCPs)

	Latitude (deg)	Longitude (deg)	Height (m)	Latitude (deg)	Longitude (deg)
	Estimated	Estimated	Estimated	Estimation Error	Estimation Error
Model Point	23.073214	72.707679	0.275406	-0.000394	0.000481
Model Point	23.072129	72.676790	0.275381	-0.000239	0.000370
Model Point	23.070998	72.683987	0.275356	-0.000158	0.000473
Model Point	23.055966	72.682827	0.275016	0.000224	0.000483
Model Point	23.049266	72.684239	0.274865	0.000324	0.000421
Model Point	23.046081	72.688828	0.274793	0.000239	0.000522
Model Point	23.040981	72.684152	0.274678	0.000179	0.000358
Model Point	23.024966	72.682741	0.274317	-0.000546	0.000159

	Latitude (deg)	Longitude (deg)
Mean at check points	-0.000046	0.000408
RMS at check points	0.000312	0.000422
Std. Dev at check points	0.000330	0.000116

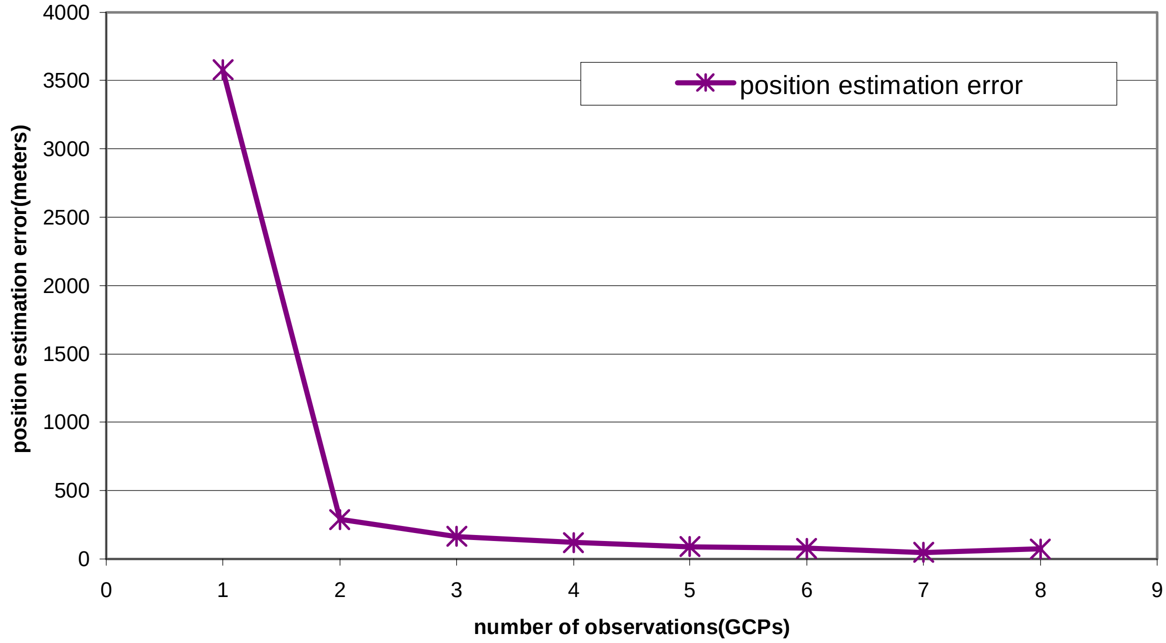

Figure 2 : POSITION ESTIMATION ERROR VS NO. OF OBSERVATIONS(NO OF GCPs) USED