High Accuracy Attitude Determination Methods for Cartographic Applications

H. R. Rao, A. Solaiappan and **R. Pandiyan**
Flight Dynamics Division, ISRO Satellite Centre, Bangalore - 560 017
hrrao@isac.ernet.in

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Abstract

Presently Attitude Determination (AD) for remote sensing satellites are done on ground using Earth Sensors, Sun Sensors, Gyros and small field of view (FOV) star sensors. The accuracy achieved on ground is of the order of 0.1 deg using earth sensor and 0.02 - 0.05 deg with narrow FOV star sensor. As the future remote sensing satellites are expected to be agile, there is a requirement that the star sensors perform even during attitude maneuvers. For cartographic applications, better location accuracy (which is in turn function of attitude determination) for the data products is expected. In this paper, the various ways and means that this could be achieved is elaborated. Some of the experiences that enhance attitude accuracy have been detailed along with results. It is expected that once such measures are taken up, the attitude determination accuracy on ground will be extremely accurate.

Overview

1. Introduction

Earth Observation Systems (EOS) such as Cartographic Satellites (CARTOSAT) require very stringent Attitude Determination (AD) accuracy due to the nature of the payloads used onboard and the applications of them on ground. Autonomous AD using onboard processors has come to stay as it eases operational complexities. However, it is not possible to account for every conceivable error sources while performing attitude control onboard. In this context, it is imperative that a ground orbit and AD processes have been an absolute necessity that aids the onboard AD process to achieve the necessary accuracy by pinpointing and quantifying various errors. This paper is aimed at listing out the sensor, attitude and orbit error sources that can be quantified by the ground processing attitude and other packages. The various error sources that are to be accounted are enumerated in the following:

a. Star CCD Error Sources: Usually, the scenario for AD has been assumed to be a set of Star Sensors together with 3-axis gyroscopes and within that context the error sources that can be estimated has been elaborated. Firstly, to obtain a better AD accuracy, the modeling should start from the star sensor itself. Star sensor is used to measure the position of star in the plane perpendicular to the boresight that is tangent to the celestial sphere at the sensor boresight. To convert the position co-ordinates from the tangent plane to angles on the celestial sphere, coefficients supplied by the manufacturers / designers are used. It has been found that these coefficients are functions of (i) distortions due to position in the Field of View (FOV) (ii) temperature (iii) star intensity and (iv) the magnetic field. To add to this, further systematic errors have also been observed in-flight and therefore, additional calibrations are required to reduce these errors significantly.

b. Errors in Star Position: It is stated elsewhere that the star sensor can give an accuracy of better than 1 arcsec (Liebe, 1995) if all the associated errors are accounted appropriately. One of the major sources of error that can arise can be from the source star catalog itself. In reality, a variety of Source Star Catalogs are available; viz. (i) Smithsonian Astrophysical Observatory (SAO) catalog, (ii) Henry Draper (HD) catalog, (iii) Durchmusterung (DM) catalog and SKYMAP (Anon., 1999) catalog, to note a few. Further corrections may be required (Roy, A. E., and Clarke, D., 1982, McCarthy, 1996) if one need to account for errors like (i) aberration, (ii) precession, (iii) nutation and (iv) proper motion. The equinox and the epoch of the various standard catalogs are also in variance and they need to be made uniform. Corrections are required for position as well as proper motion of stars that are catalogued in the various source catalogs for true of date. In this connection, SKYMAP SKY2000 catalog has been designed to facilitate the incorporation of data critical to the support of future space missions that use chargecoupled device (CCD) star trackers (CCDST) for attitude determination. The methods to remove these errors will bring about a sea change in the accuracy of AD processes.

c. Gyro Calibration: An attitude accuracy study for EOS has been provided in (Lesikar II and Garrick, 1996) wherein the extent of accuracy to which the attitude sensor and gyro calibrations can be performed is determined. The major sources of error in a gyroscope can be attributed to (i) rate bias errors (deg/sec), scale factor errors (dimensionless), (iii) misalignment errors (deg) and (iv) gyro noise. In-flight gyroscope calibration has been often performed with one of the variations of the Davenport Gyroscope Calibration Algorithm (Welter, G. *et al*, 1996, Cray, C. W., *et al*, 1990). Methods have been developed for determining adjustments to drift rate bias vector, a scale factor and alignment transformation matrix. The algorithms are envisioned as applied in a batch mode least squares algorithm and most suitable for ground processing only. Further, maneuvers are required to observe the entire gyro biases, misalignments and scale factor errors. Also, complete observability can be achieved only when the maneuvers are about each individual axis. Such a requirement does not allow onboard computation of errors induced due to scalefactor and misalignment in gyroscopes due to Kalman Filtering. Hence ground AD is expected to provide better estimates of biases, scale factor etc.

d. Misalignment Estimation / Alignment Calibration: Several algorithms have been developed to determine the in-flight alignments of attitude sensors. The algorithms work by comparing the dot products of simultaneously measured unit vectors in spacecraft coordinates to the dot products of the corresponding unit vectors in reference coordinates (Shuster, M. D. *et al* 1991a, Shuster, M. D. *et al*, 1991b). Similarly, details of several Kalman Filter - based algorithms have been elaborated in (Pittelkau, 2000 & Lee, M. 1995). In a given mission scenario, which algorithm needs to be used depends upon the accuracy requirement. The accuracy of this alignment algorithm procedure is limited by the sensor measurement noise. However, if large amount of data is provided, the effect of noise will be adequately reduced. In the ground AD, which is a post facto mode, since all the data are available it is expected that the in-flight alignments could be estimated more accurately.

e. AD Estimation Procedures - Effect of No. of Stars: AD using star sensor precedes successful star identification. Star identification algorithms are numerous and basically divided into two categories; (i) the first category is based on angular separation between star pairs and (ii) the second category using star patterns such as triplets, quartets etc. When the expected stars are greater than or equal to 7 in the Field of View (FOV) of the sensor, then star identification procedures based on angular distances between star-pairs is best suited. On the other hand, for narrow FOV star sensors or for sensors where the expected number of stars are greater than 3 but less than 4, identification algorithms based on star patterns are best suited. Interestingly, the AD accuracy is greater if the number of stars identified / frame is the largest.

A new algorithm has been developed for cartographic applications based on angular distance between stars, which is a fast and robust star identification technique that does not use a searching phase, thus highly suitable for onboard implementation (Mortari, 1997). The algorithm uses the angular separation information only and does not require magnitude information as well as a priori attitude information. The algorithm has two parts namely, a K- vector Star Pair Identification Technique (SPIT) and Reference-Star Star-Matching Identification Technique (SMIT). The SPIT algorithm is responsible for search-less identification of stars. The principal idea of the proposed method is based on the fact that any star direction can always be expressed as a linear combination of two non-parallel star directions together with their vector cross product.

f. Orbit & Attitude Adjustment using Ground Control Points (GCP): Orbit adjustment is the first step for geometric correction of cartographic images. This approach requires a large number of GCPs. Least squares filtering (Internet downloaded (a)) is used to minimize the errors between the observation vectors provided by the satellite and the known GCP vectors corresponding to these vectors. In this process, the minimum number of GCPs required can also be arrived at efficiently. The attitude data at the ground station includes discrete samples of the attitude quaternions and gyro rate measurements for some duration. A post processing Kalman filter (Internet downloaded (b), 1998 & Itzhack, 1996) of the attitude data stream is used to incorporate all the measurement information. The output of the filter, which is a smoothed data stream, will be a new attitude data stream representing the combined gyro, quaternion and gyro drift data. This smoothed data stream will represent the low frequency information of the satellite, which is accurate without noise effect.

In the following, methods to tackle above error sources for cartographic missions are elaborated. When first-hand experience is available, the details along with past experience are provided. In case of new additions, the algorithm details are highlighted.

2. Star CCD Error Modeling

The systematic errors in star sensor measurements appear (i) during conversion of sensor output into an observation vector in the sensor frame and (ii) during transformation of the observation vector from sensor frame to the attitude reference frame (Anon., 1999). The first one is known as transfer function calibration and determines the constants needed to convert sensor output into physical quantities. The second one is the well-known misalignment estimation or alignment calibration procedure, which appears as part of attitude angles themselves.

Transfer Function Calibration / FOV Calibration

One of the FOV calibration coefficient is what is known as scale factor $\mathbf{k}$ that connects angles in celestial sphere ϕ and θ to position coordinates H and V as

$$\begin{aligned} \phi &= \mathbf{k}\,\mathrm{H} \\ \theta &= \mathbf{k}\,\mathrm{V} \end{aligned} \qquad (2.1)$$

where $\mathbf{k}$ is the scale factor provided by the manufacturer.

The remaining calibration coefficients are established using linear regression method or Lavenberg-Marquardt method by assuming suitable models. The models are linear functions of the observed FOV angles to the expected FOV angles. For example, the GRO/UARS mission considered a model (Liebe, 1995) of the form

$$\begin{aligned} \phi' &= \phi + c_1\phi + c_2 + c_3\theta \\ \theta' &= \theta + d_1\theta + d_2 + d_3\phi \end{aligned} \qquad (2.2)$$

where c_1 and d_1, are the adjustments to the horizontal and vertical scale factors. Note that, c_k and d_k, k=2,3 are adjustments which correspond to small rotational adjustments that include the effects of attitude error. For another mission EUVE, a much higher polynomial model is used as shown below.

$$\begin{aligned} \phi' &= \phi + c_1 + c_2\phi + c_3\phi^2 + c_4\,\phi^3 + c_5\theta + c_6\phi\theta \\ \theta' &= \theta + d_1 + d_2\theta + d_3\theta^2 + d_4\theta^3 + d_5\phi + d_6\phi\theta \end{aligned} \qquad (2.3)$$

where c_k and d_k, k=1,6 are the parameters to be determined. In yet another work (Roy, A. E., and Clarke, D., 1982), the position co-ordinates H and V themselves are corrected for various errors using a polynomial with 19 coefficients. The model is of the form

$$\begin{aligned} \mathrm{H}' = {} & c_1 + c_2\mathrm{H} + c_3\mathrm{H}^2 + c_4\mathrm{H}^3 + c_5\mathrm{V} + c_6\mathrm{V}^2 + c_7\mathrm{V}^3 + c_8\mathrm{X} + c_9\mathrm{X}^2 \\ & + c_{10}\,\mathrm{VH} + c_{11}\mathrm{VX} + c_{12}\mathrm{HX} + c_{13}\mathrm{V}^2\mathrm{H} + c_{14}\mathrm{V}^2\mathrm{X} + c_{15}\mathrm{H}^2\mathrm{V} + c_{16}\mathrm{VX}^2 \\ & + c_{17}\mathrm{VHX} + c_{18}\,\mathrm{H}^2\mathrm{X} + c_{19}\mathrm{X}^2\mathrm{H} \\ \mathrm{V}' = {} & d_1 + d_2\mathrm{H} + d_3\mathrm{H}^2 + d_4\mathrm{H}^3 + d_5\mathrm{V} + d_6\mathrm{V}^2 + d_7\mathrm{V}^3 + d_8\mathrm{X} + d_9\mathrm{X}^2 \\ & + d_{10}\,\mathrm{VH} + d_{11}\mathrm{VX} + d_{12}\mathrm{HX} + d_{13}\mathrm{V}^2\mathrm{H} + d_{14}\mathrm{V}^2\mathrm{X} + d_{15}\mathrm{H}^2\mathrm{V} + d_{16}\mathrm{VX}^2 \\ & + d_{17}\mathrm{VHX} + d_{18}\,\mathrm{H}^2\mathrm{X} + d_{19}\mathrm{X}^2\mathrm{H} \end{aligned} \qquad (2.4)$$

where c_k and d_k, k=1,19 are the parameters to be determined. These equations are applied successively with the value of X being first temperature, then star intensity and then each of the three components of the magnetic field. Thus this model provides adjustments to bias, scale factor, quadratic and cubic nonlinearities and the coupling between the axes. Later a combined scale factor $\mathbf{k}$ is used on the lines of Equation (2.1) to convert the position coordinates into angular coordinates.

It is important to note that the FOV calibration is done by using in-flight data and on ground using ground AD process. Further, if the CCD arrays are getting affected due to aging, the calibration process need to be repeated often. In a nutshell, the various other minor sources that can contribute AD error through Star CCD camera can be enumerated as follows.

1. The digital resolution - the number of bits to represent the centroid information. A typical value for NASA Star Tracker (BECD) is 7.78 arc sec
2. Random errors in the position measurements depending on the star intensity and position in the FOV. A typical range from literature is 8-24 (1s) arc sec.
3. Residual systematic calibration errors. A typical value is <10 arc sec (1s)
4. Time uncertainty on star sensor measurement can effect rate dependent error

when the spacecraft is in maneuvers. A typical range is 8-24 arc sec.

3. Errors in Star Position and the Modeling

Before one would like to utilize the star catalog position information, variety of corrections need to be applied for the positional data to get appropriate accuracy. The major corrections required are (i) aberration and parallax error caused by earth's motion about the Sun, (ii) precession and nutation effect due to sun-moon attractions of earth and (iii) the proper motion correction due to the star's own velocity with respect to the Sun. In the following the definitions of various errors and the mathematics required to correct them appropriately have been discussed.

a. Stellar Aberration: The apparent shift of the brighter, presumably nearer, stars due to the earth's annual journey in its heliocentric orbit is called stellar aberration. This apparent direction in which the star is seen is depending upon the direction of the velocity of light from the star and the velocity of the earth in its orbit.

The typical value for a given star is given by

$$\Delta\Theta = 206.265\frac{v}{c}\sin\Theta = \kappa\sin\Theta = 20.''496\sin\Theta \quad \text{.. (3.1)}$$

where v = Earth's orbital velocity in km/s (30 km/s) and c = velocity of light in km/s (299792.5 km/s) and Θ is the angle between the star's direction and the instantaneous direction in which the earth is travelling. In case of an artificial earth satellite with an orbital velocity of 7 km/s, the aberration in its observed position is of the order of 5 arc sec and must be corrected for precise tracking. In evaluating the combined effect of earth-satellite aberration, one should use the direction of the relative velocity of the combined system instead of the direction of the earth velocity.

b. Parallax Errors: The apparent shift in the position of a planet subtended due to the earth disc in the backdrop of fixed stars provides the horizontal parallax error. Similarly, the apparent shift in the position of stars due to the orbital path of the earth with respect to sun is known as stellar parallax. The annual parallax changes the coordinates of a star, but the changes are so small (order of x < 1 arcsec) that they are discarded in problems. For example, the largest annual parallaxes are computed for Proxima Centauri, 0. 762 arcsec and α Centauri, 0.756 arcsec.

c. Precession of Equinoxes: The fact that the earth is an oblate spheroid with an equatorial bulge and both Sun and moon move in apparent orbits inclined to the plane of the earth's equator, their net gravitational attractions act along the lines joining these bodies to the earth's centre. This action would produce a tendency to tilt the earth's rotational axis. The north celestial pole therefore traces out a small circle of radius ε with the precessional period of 26000 years. This produces a continuous movement of First point of Aries Γ along the ecliptic at a rate of 50''.2 per annum and a corresponding movement of the celestial equator.

In this development, the orientation of the mean equator and equinox of date T with respect to the equator and equinox of 2000.0 is defined by three-precession angles.

$$\begin{aligned}
\varsigma &= 2306''.2181\,T + 0''.30188\,T^2 + 0''.017998\,T^3 \\
\vartheta &= 2004''.3109\,T + 0''.42665\,T^2 + 0''.041833\,T^3 \\
z &= 2306''.2181\,T + 1''.09468\,T^2 + 0''.018203\,T^3 \quad \text{...................................... (3.2)}
\end{aligned}$$

where T is measured in Julian centuries from 2000.0 as

$$T = (JD - 2451545.0) / 36525.0$$

with JD being the Julian Date of the date. The transformation from position coordinates r_{2000}, referring to the mean equator and equinox of 2000, to coordinates w.r.t. the mean equator and equinox of date T may now be written as

$$r_{ToD} = P\, r_{2000}$$

where the precession matrix P is given as the product of three consecutive rotations. The suffix ToD represents true of date. The P matrix is given by

$$P = \begin{pmatrix} -\sin z\sin\zeta + \cos z\cos\vartheta\cos\zeta & -\sin z\sin\zeta - \cos z\cos\vartheta\cos\zeta & \cos z\sin\vartheta \\ \cos z\sin\zeta + \sin z\cos\vartheta\cos\zeta & \cos z\cos\zeta - \sin z\cos\vartheta\sin\zeta & -\sin z\cos\vartheta \\ \sin\vartheta\cos\zeta & -\cos\vartheta\cos\zeta & \cos\vartheta \end{pmatrix} \text{.. (3.3)}$$

d. Nutation: Nutation, a slight oscillatory motion of the Earth's axis is accompanied by precession. It has a maximum period of 18.6 years and with axes of the deviation ellipse about 18 arc sec to 14 arc sec on either side. This effect is due to the minute wobble of the Earth's axis of rotation caused by the Moon's action on the Earth's equatorial bulge. Similar to precession correction, for nutation correction one uses the transformation

$$r_{ToD} = N\, r_{MoD}$$

where $N = R_x(-\varepsilon-\Delta\varepsilon)\, R_z(-\Delta\Psi)\, R_x(\varepsilon)$. Either a Fourier coefficient or Chebyshev polynomial fit approach (Cray, C. W. *et al*., 1990) to compute nutation angles $\Delta\Psi$ and $\Delta\varepsilon$ is followed. The elements of the transformation matrix N in equatorial coordinates are

$$N = \begin{pmatrix} \cos\Delta\Psi & -\cos\varepsilon\sin\Delta\Psi & -\sin\varepsilon\sin\Delta\Psi \\ \cos\varepsilon'\sin\Delta\Psi & \cos\varepsilon\cos\varepsilon'\cos\Delta\Psi+\sin\varepsilon\sin\varepsilon' & \sin\varepsilon\cos\varepsilon'\cos\Delta\Psi-\cos\varepsilon\sin\varepsilon' \\ \sin\varepsilon'\sin\Delta\Psi & \cos\varepsilon\sin\varepsilon'\cos\Delta\Psi & \sin\varepsilon\sin\varepsilon'\cos\Delta\Psi+\cos\varepsilon\cos\varepsilon' \end{pmatrix} \quad .... (3.4)$$

where $\varepsilon' = \varepsilon + \Delta\varepsilon$ and ε is the mean obliquity of the ecliptic at time T given by

ε = $23^0.43929111 - 46''.8150T - 0''.00059T^2 + 0''.001813\,T^3$

e. Proper Motion: Proper motion is defined as the annual angular drift in its heliocentric direction on the celestial sphere due to its space velocity relative to Sun. The drifts are then divided by the number of years between the epochs to obtain the star's proper motion in right ascension and declination. If X is the position of the star at a certain epoch with right ascension and declination (α,δ) and Y is the position of the star after a year. The great circle arc XY is the star's annual proper motion represented by μ. The components of m in right ascension and declination are represented as μ_α and μ_δ can be derived as

$$\mu_\alpha \cos\delta = \mu \sin\theta \quad \text{and}$$
$$\mu_\delta = \mu \cos\theta$$

In practice, μ_α is usually expressed in seconds of time per annum while μ_δ is expressed in seconds of arc per annum.

For lengths of time up to several hundreds of years, proper motion corrections can be applied linearly, as follows:

$$\alpha_I = \alpha \quad + \mu_\alpha\, \Delta t + \left[\frac{1}{2}\Delta\mu_\alpha\, \Delta t^2\right]$$
$$\delta_I = \delta \quad + \mu_\delta\, \Delta t + \left[\frac{1}{2}\Delta\mu_\delta\, \Delta t^2\right]$$

where the terms in square brackets represent error terms because of neglecting the secular acceleration ($\Delta\mu_\alpha$, $\Delta\mu_\delta$) in proper motion and where (α_I, δ_I) right ascension and declination with proper motion corrected to epoch I and (α_J, δ_J) right ascension and declination at standard catalog epoch J.

4. Misalignment Estimation / Alignment Calibration

Relative misalignments between a pair of vector star sensors can be computed using a maximum likelihood estimator technique. The algorithm works on the principle of minimizing the cosine of included angles between the reference vectors and the observation vectors. Normally, if the reference vector is same as the observation vector, there is no attitude / alignment error. The details are as given below.

Details of Measurement Model

The observed unit vector in the sensor frame at any given instant is given by

$$\hat{u}_{i,k} = \hat{u}^t_{i,k} + \Delta\hat{u}_{i,k} \qquad (4.1)$$

where $\hat{u}_{i,k}$ is the actual measurement, $\hat{u}^t_{i,k}$ is the true measurement supposed to be and $\Delta\hat{u}_{i,k}$ is the error vector in the sensor measurement, respectively. It is to be noted that the suffix i = 1,2 corresponds to the number of sensors, the suffix k correspond to a temporal index. $\Delta\hat{u}_{i,k}$ is assumed Gaussian, zero mean and white with covariance matrix R_{uik}. The measurement model referred here is known as QUEST measurement model. The covariance matrix R_{uik} has been derived and found to be in the form

$$R_{uik} = \sigma_i^2\,[\,I - \hat{u}_{i,k}\,\hat{u}^t_{i,k}]$$

where σ_i^2 is the variance of the measured misalignment angles with respect to sensors. Now, let S_i be the alignment matrices of vector sensors, i = 1, 2 respectively. Then, the measured unit vector in the body frame can be written as

$$\hat{n}_{i,k} = S_i\,\hat{u}_{i,k} = S_i\,[\hat{u}^t_{i,k} + \Delta\hat{u}_{i,k}\,] = \hat{n}^t_{i,k} + \Delta\hat{n}_{i,k} \qquad (4.2)$$

where the corresponding covariance matrix is given by

$$R_{vik} = \sigma_i^2\,[\,I - \hat{n}_{i,k}\,\hat{n}^T_{i,k}]$$

and the $\Delta\hat{n}_{i,k}$ is again Gaussian, zero mean and white.

Let the apriori alignment matrix be S_i where i = 1, 2 depending upon the sensor head. Now, a relation connects misalignment matrix and a priori alignment matrix as given below

$$S_i = M_i\,S_i^o \qquad (4.3)$$

where S_i^o is the alignment matrix that was known a priori and

$$M_i = \begin{bmatrix} 1 & -\theta_{zi} & \theta_{yi} \\ \theta_{zi} & 1 & -\theta_{xi} \\ -\theta_{yi} & \theta_{xi} & 1 \end{bmatrix}$$

is the misalignment matrix and θ_i are the misalignment angles of the sensors.

The uncalibrated body-referenced observation vector is given by

$$\hat{u}^{o}_{i,k} = S_i^{o}\, \hat{U}_{i,k} \qquad (4.4)$$

Using equations (4.2) and (4.3) in (4.1) we get,

$$\hat{U}_{i,k} = S_i^{oT} M_i^{T} \hat{u}^{t}_{i,k} + \Delta\hat{U}_{i,k} \qquad (4.5)$$

Let the reference vector in orbit reference frame be given by

$$\hat{V}_{i,k} = \hat{V}^{t}_{i,k} + \text{Zero Noise}$$

and the measured direction in body frame now is

$$\hat{u}^{t}_{i,k} = A_k \hat{V}_{i,k}$$

where A_k is the attitude matrix at a given time.

After some mathematical simplifications one can obtain

$$\hat{u}^{o}_{i,k} = \{ \mathbf{I} - [[\theta_i]] \}\hat{u}^{t}_{i,k} + \Delta\hat{u}^{o}_{i,k} \qquad (4.6)$$

Since the heads are mounted on a rigid structure (Figure 4.1), the attitude rotation does affect the measurement vector and the reference vector in a similar way. Therefore, the dot product between the observed vectors and reference vectors will have similar effect. The difference between the included angles thus obtained between the observed and referenced vectors will be devoid of the effect of attitude of the spacecraft.

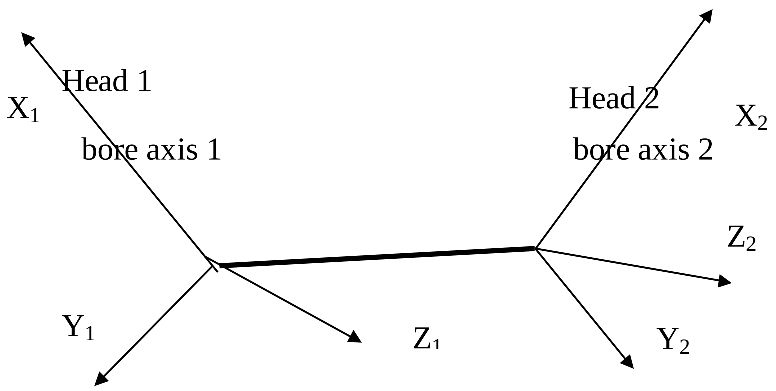

Figure 4.1: Definition of Sensor Axes

Now, taking the dot product between the observed vectors we get,

$$\hat{u}^{o}_{i,k} \bullet \hat{u}^{o}_{i,k} - V_{i,k} \bullet V_{i,k} = \{\hat{u}^{o}_{i,k} \times \hat{u}^{o}_{i,k}\} \bullet (\theta_{\mathbf{i}} - \theta_{\mathbf{j}}) + \Delta Z_k \qquad (4.7)$$

where $\theta_{\mathbf{i}}$ and $\theta_{\mathbf{j}}$ are the misalignment vectors of head1 and head2, respectively and ΔZ_k is the noise equivalent of the measurement constructed with following properties.

$$\Delta Z_k = \hat{u}^{t}_{i,k} \bullet \Delta\hat{u}^{o}_{j,k} + \hat{u}^{t}_{j,k} \bullet \Delta\hat{u}^{o}_{i,k} \qquad (4.8)$$

$E(\Delta Z_k) = 0$ and $E(\Delta Z_k^2) = P_{zk}$ = covariance matrix of measurements.

The equation (4.7) can be rewritten as

$$Z_k = \{\hat{u}^{o}_{i,k} \times \hat{u}^{o}_{i,k}\}^T \Psi + (\Delta Z_k) = H_k \Psi + (\Delta Z_k) \qquad (4.9)$$

where $\Psi = (\theta_{\mathbf{i}} - \theta_{\mathbf{j}})$. The application of maximum likelihood estimation procedure to equation (4.9) gives rise to

$$(P^{-1}_{\Psi\Psi})\, \Psi = \sum H_k^T P^{-1}_{zk} Z_k \qquad (4.10)$$

and

$$P^{-1}_{\Psi\Psi} = \sum H_k^T P^{-1}_{zk} H_k \qquad (4.11)$$

The matrix $(P^{-1}_{\Psi\Psi})$ is known as the covariance matrix of the relative alignment of the sensor.

Case Study - Preliminary Estimation of Misalignments of Star Sensor for TES

It is assumed that the head 1 is mounted perfectly and does not change due to launch shock and vibration. The star sensor mounting angles are given in Table 4.1.

Table 4.1: Mounting angles of TES Star Sensor Heads (LEOS)

Heads	MOUNTING ANGLES WRT MRC (in deg)		
	rotation about 2	rotation about 3	rotation about 1
1	224.9460	35.3119	-119.8430
2	224.9185	-35.6943	-59.92240

The converged final mounting angle of the head2 sensor is given as (224.9220, -35.6894, -59.9224) degrees.

5. AD Estimation Procedures - Effect of Number of Stars

It is proven that an increase in the number of stars in the star sensor identification can improve the AD accuracy. This requires Wider Field of View (WFOV) star sensors. Further, WFOV star sensors guarantee continuous sightings of stars and thereby continuous attitude information. A new identification method and AD processor has been developed for cartographic applications which is working based on Lost-in-Space (LIS) concept. The important steps / details of the algorithm and the first hand experience of this package for GSAT-1 (due to 5 star availability) has been provided in the following:

5.1 Star-pair catalog generation: This procedure is to identify a small set of likely catalog star-pair from the master star catalog, based on their angular separation. The output of this step will be the basis of the 'star matching' phase. Let v_i & v_j be the unit vectors of two stars in inertial frame from the master star catalog. These stars will be within the star sensor field of view, if and only if the angle between the vectors is less than star sensor field of view angle *(ϑ_{FOV}).* The inner product of the vectors is greater than the cosine of ϑ_{fov}.

$$\left(v_i \cdot v_j\right) \geq \cos\left(\vartheta_{FOV}\right) \quad \ldots\ldots\ldots \; (5.1)$$

Any two stars satisfying the above condition is called 'admissible star pair' or simply 'star-pair' with respect to the sensor field of view.

$$Cos\,(\vartheta_{FOV}) \;\; \leq \;\; SP(k) \;\; \prec \;\; SP(l) \;\; \leq \;\; 1.0$$

The admissible star-pair objects are sorted in ascending order based on the angle *(ϑ_{ij}).* All the characteristics of star-pair are moved along with the objects when it is sorted.

5.2 Star-Pair Identification Technique: Once the angular separation for any measured star-pair is known, it is possible to locate the desired object directly in a simplified process by avoiding many index checks. A straight line connecting the two extremes has on the average only one element for step. A steeper straight-line that connects the two points is found to be an excellent approximation to locate the desired element. The k-vector star-pair identification procedure starts by accessing the candidate set of cataloged stars for each measured star pair. For any observed star-pair, a number of n admissible star-pairs, which can be arranged to form the n dimensional index arrays *[i,j],* are obtained from star catalog. The dimension of n is to be determined by how many star pairs exist such that inter star sine angle of each star pair falls within the measured value, plus or minus the sensor accuracy.

5.3 Star Matching Identification Technique: This procedure identifies the correspondence between the angular structures of the observed stars and those build from the likely detected ones by the star-pair identification phase. Once the admissible star-pair sets have been calculated, there arises the problem of identifying which stars are the observed ones. This is accomplished here with the reference star matching identification method. Suppose we have finite number of measured stars, say five stars for example. We choose a star and call it the pole star. The remaining measured stars are called satellite stars. To this pole star and its satellite star's angular distances, we have four sets of k-vector stars. If a star from the catalog is a possible candidate for the measured pole star, it should be a member of at least one pair in each of the sets. Thus, it should belong to the intersection of these sets.

Once identification is over, QUEST algorithm is used to obtain accurate attitude estimation from vector observations. The Star Sensor Attitude Determination package Quaternion Attitude In Lost in Space (QUALIS) based on Searchless Angular Separation algorithm is tested in post-launch scenario after the successful launch of GSAT spacecraft. The experience gained in the various phases of operation is elaborated in the following sections.

5.4 Transfer Orbit - Gyro Calibration (GYROCAL) Phase: In this phase, final attitude angles are deduced in Inertial Sun Pointing Reference Frame. Subsequent to putting into Transfer Orbit, the star sensor was switched on immediately. After some tuning, the star sensor provided useful 5 star data. Then onwards the QUALIS package was used to obtain attitude as though the attitude was not known for the T.O. phase. The output of these packages were compared with Earth Sensor (ES) and Digital Sun Sensor (DSS) outputs. The following observations were made. Initially, the spacecraft has been given a roll bias of -10 deg to extend the visibility of ES for Gyro Calibration purposes. Since the spacecraft is Sun pointing and controlled with respect to DSS output, a roll error of -10.0 deg is shown by DSS as well as Star Sensor outputs. Further, it is noted that the yaw error computed using star sensor has matched yaw error shown by DSS output.

On April 23rd, 2001 the spacecraft roll bias has changed in between from -10 deg to 0.0 deg in T.O. Orbit. In order to test the package in Lost-in-Space Mode in a true sense, it is decided to compute attitude immediately after AMF is completed during which period the spacecraft is in Inertial Reference Unit (IRU) control. Prior to AMF, the IRU is reset to 0 deg roll error with respect to a -10 deg roll bias of the spacecraft. After the AMF fire, the IRU still treats the -10 deg roll bias as zero and when the roll error is made to zero, this is shown as +10 deg roll error in IRU outputs. The spacecraft roll error is made to zero around 2001-04-23-01-30-00-000 hours and around this time the IRU output shows a +10 deg roll bias. This shows that the package is capable of computing attitude even in lost-in-space mode.

5.5 Synchronous Orbit (SO) Phase: In this mode of operation, the spacecraft attitude is computed in the Orbit Reference Frame (ORF) for easy comparison. The spacecraft is three-axis stabilized with a mechanical pitch bias of +3.0 degree during this phase. The FRSS has sighted 4-5 stars at this juncture as well. The star sensor was used to obtain attitude at this juncture to get near S. O attitude values. The analysis is performed with Searchless Angular Separation algorithm, for 24th April 2001. A pitch error around +3 deg and yaw error around -1.2 deg is noticed from the outputs. Since the spacecraft had been provided a mechanical pitch bias of +3 degrees for station pointing purposes, the FRSS reflects a

+3 degree pitch error. It is also confirmed from Controls team that the yaw error is of the order of -1 deg.

It has been shown elsewhere (Mortari, D., *et al*, 2000), for a star sensor with a noise level of 0.025 deg (3σ), a three star identification to 10 star identification improves the attitude accuracy by 0.01 degree approximately.

6. Orbit & Attitude Adjustment using Ground Control Points (GCP)

Further improvement in the accuracy can only be obtained by using GCPs in order to adjust orbit and attitude. A least squares adjustment procedure for orbit adjustment is envisaged. The observation vectors from the satellite may not pass through the corresponding GCP vector due to errors in the on-board data. The correction of orbit from (x_0,y_0,z_0) to (x,y,z) may be performed using

$$
\begin{aligned}
X_i &= x(t_i) - S_i\, U_{xi} \\
Y_i &= y(t_i) - S_i\, U_{yi} \\
Z_i &= z(t_i) - S_i\, U_{zi} \\
x(t_i) &= x_0 + a_0 + a_1.t_i + a_2.t_i^2 \\
y(t_i) &= y_0 + ab_0 + b_1.t_i + b_2.t_i^2 \\
z(t_i) &= z_0 + c_0 + c_1.t_i + c_2.t_i^2
\end{aligned}
\qquad \text{..........} \; (6.1)
$$

where X_i, Y_i and Z_i are object co-ordinates of GCP; u_x,u_y,u_z are components of the observation vector, $x(t_i)$, $y(t_i)$ and $z(t_i)$ are satellite position after correction; x_0, y_0 and z_0 are satellite position before correction and a_i, b_i, and c_i (i=0,1,2) are coefficients for orbit correction, t_i represents sampling time and S_i is the scale factor matrix.

For the purpose of correcting images, one need not distinguish between orbit and attitude errors. Letting either orbit or attitude correction absorb the existing errors will correct the image. For payload misalignment estimation, that is to separate the ephemeris error from the attitude error, one should use the most precise ephemeris to start with. Further filtering can also be thought of for improvement in position by suitably providing weight factor for observation and apriori information.

7. Concluding Remarks

Ground AD process has the capability to obtain (i) total transfer function calibration of Star CCD Camera (ii) alignment estimation using batch or Kalman Filter algorithms (iii) gyro calibration (iv) to estimate effect of scale factor and misalignments in gyroscope measurements and (v) to use data from other sources like GCP. If one has to implement all these capabilities onboard, the resulting model will have large number of states that are to be estimated. The convergence of such a large filter onboard may be very difficult. If the star CCD camera and gyroscope parameters are not estimated simultaneously, calibration results due to star CCD camera have not been found shrinking. This is because the spacecraft states when propagated using the dynamically modeled attitude obtained using gyroscope measurements is found to conflict with the star CCD camera observation vectors. Batch algorithms cannot be implemented onboard, which require enormous amount of data to be stored onboard. In lieu of the above facts, the onboard AD process can never be better than ground AD process. The ground AD process is the one that can help onboard AD process to better its accuracy level by providing scalefactor, misalignment and gyro drift rate bias more accurately and this is reported very widely by other spacecraft operators. Finally, the calibration procedures and other algorithms for orbit and attitude adjustment have also been enumerated with partial application results to some of them. It can be carried out on ground processing along with adjustment procedures with GCPs for orbit/AD improvement on ground.

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